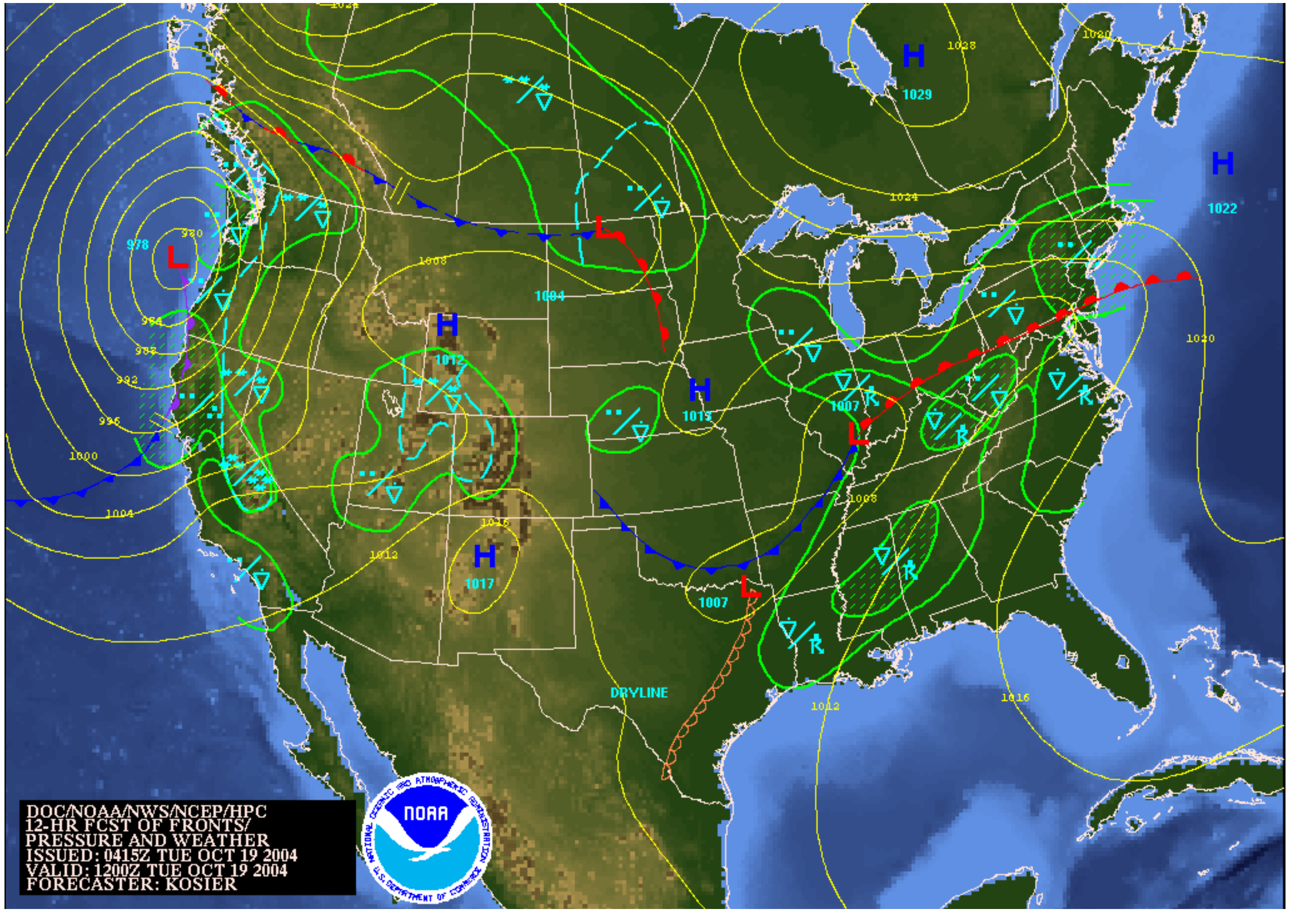
Dynamics IV: Geostrophy
SIO 210 Fall, 2015

- Geostrophic balance
- Thermal wind
- Dynamic height
- READING:
  - DPO: Chapter (S)7.6.1 to (S)7.6.3
  - Stewart chapter 10.3, 10.5, 10.6 (other sections are useful for those of you wishing more dynamics)

Example from atmosphere of nearly geostrophic flow

NWS daily weather map



- High and low pressure centers
- Winds blow around the highs and lows
- Clockwise around the highs (northern hemisphere) due to Coriolis

Coriolis effect at very short time scales? NO

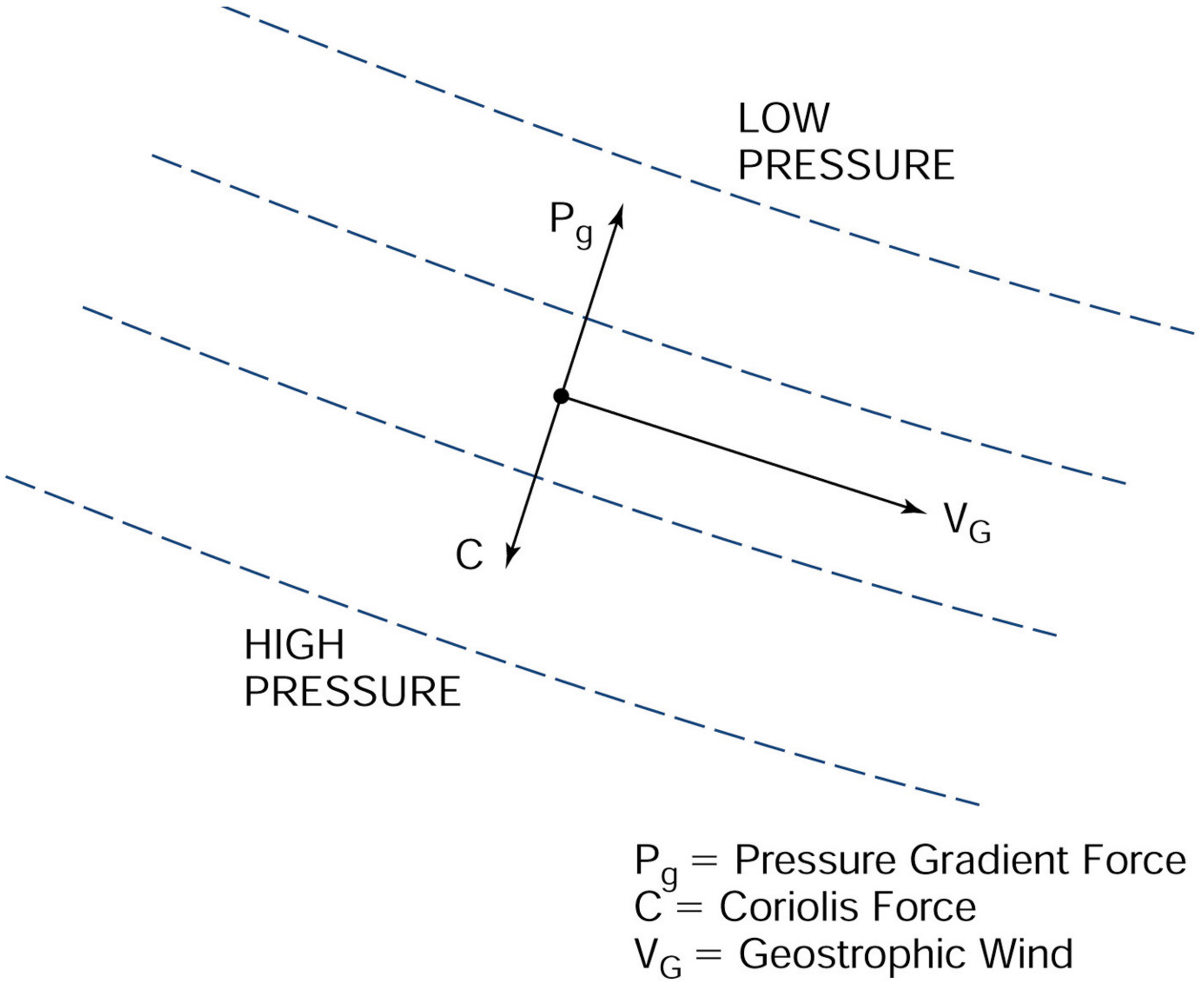
What about your bathroom sink? Can you see the Coriolis effect on the spiral of water?

NO, unless you have an extremely good laboratory!

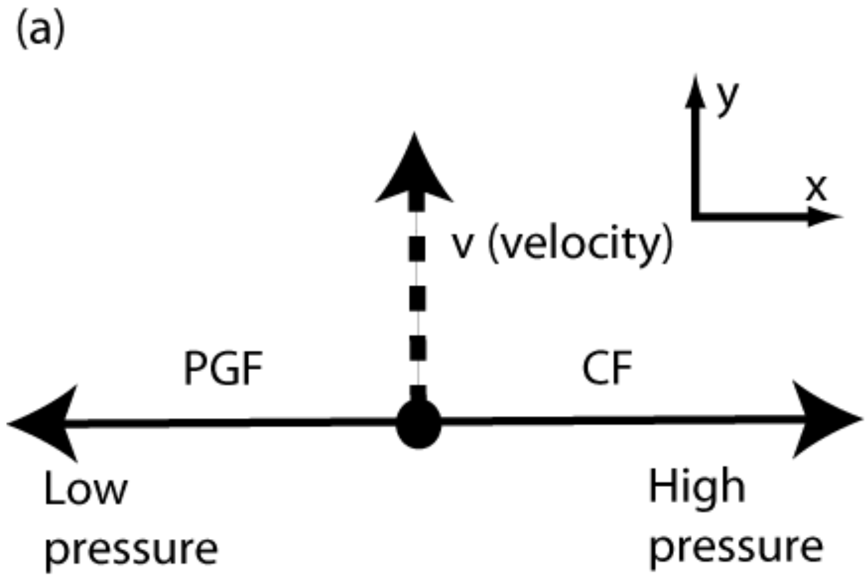
WHY? Because time scale is much shorter than 1 day (Pressure gradient force or sloshing force is balanced by acceleration, not Coriolis)

Geostrophy: PGF balanced by Coriolis force

Northern hemisphere: flow to the right of the PGF

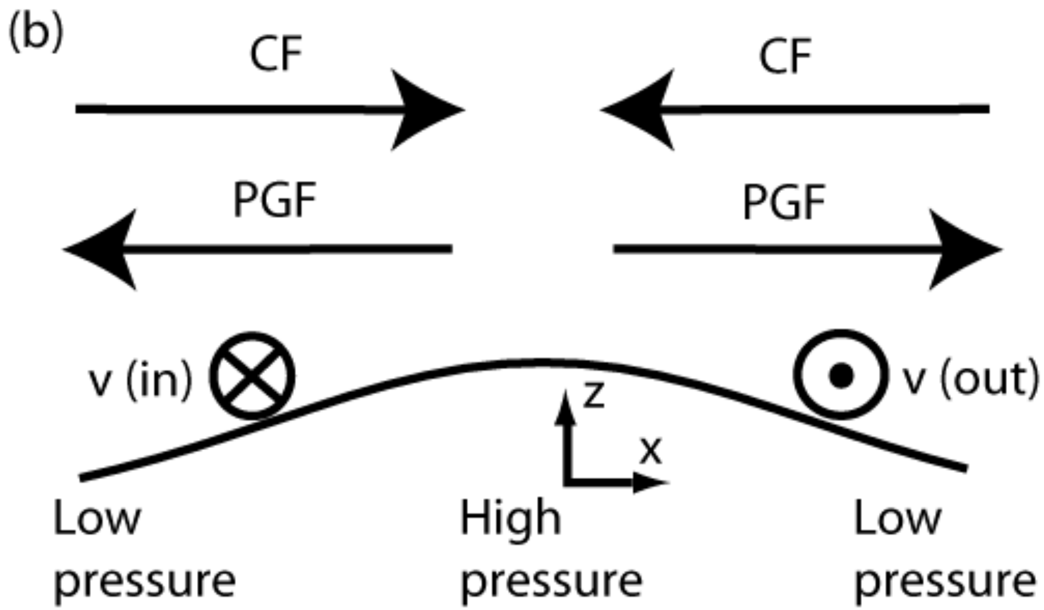


Other views



Looking down onto the ocean

(Northern hemisphere examples for velocity directions)



Looking at a cross-section

Think of ball rolling down hill from high to low pressure, turning to the right due to Coriolis

DPO Fig. 7.9

Complete force balance with rotation

Three equations:

Inertial motion

Geostrophic flow

Horizontal (x) (west-east)

acceleration + advection + Coriolis =

pressure gradient force + viscous term

Horizontal (y) (south-north)

acceleration + advection + Coriolis =

pressure gradient force + viscous term

Vertical (z) (down-up)

acceleration +advection (+ neglected very small Coriolis) =

pressure gradient force + effective gravity

(including centrifugal force) + viscous term

Final equations of motion (momentum equations in Cartesian coordinates)

Inertial motion

Geostrophic flow

x: $\partial u/\partial t + u\,\partial u/\partial x + v\,\partial u/\partial y + w\,\partial u/\partial z - fv =$
$- (1/\rho)\partial p/\partial x + \partial/\partial x(A_H \partial u/\partial x) +$
$\partial/\partial y(A_H \partial u/\partial y) + \partial/\partial z(A_V \partial u/\partial z)$ (7.11a)

y: $\partial v/\partial t + u\,\partial v/\partial x + v\,\partial v/\partial y + w\,\partial v/\partial z + fu =$
$- (1/\rho)\partial p/\partial y + \partial/\partial x(A_H \partial v/\partial x) +$
$\partial/\partial y(A_H \partial v/\partial y) + \partial/\partial z(A_V \partial v/\partial z)$ (7.11b)

z: $\partial w/\partial t + u\,\partial w/\partial x + v\,\partial w/\partial y + w\,\partial w/\partial z$ (+neglected small Coriolis) $=$
$- (1/\rho)\partial p/\partial z - g + \partial/\partial x(A_H \partial w/\partial x) +$
$\partial/\partial y(A_H \partial w/\partial y) + \partial/\partial z(A_V \partial w/\partial z)$ (7.11c)

(where g contains both actual g and the effect of centrifugal force)

Geostrophic (and hydrostatic) force balance

Three (approximate) equations:

Horizontal (x) (west-east)

Coriolis = pressure gradient force

$$-fv = -(1/\rho)\partial p/\partial x$$

Horizontal (y) (south-north)

Coriolis = pressure gradient force

$$fu = -(1/\rho)\partial p/\partial y$$

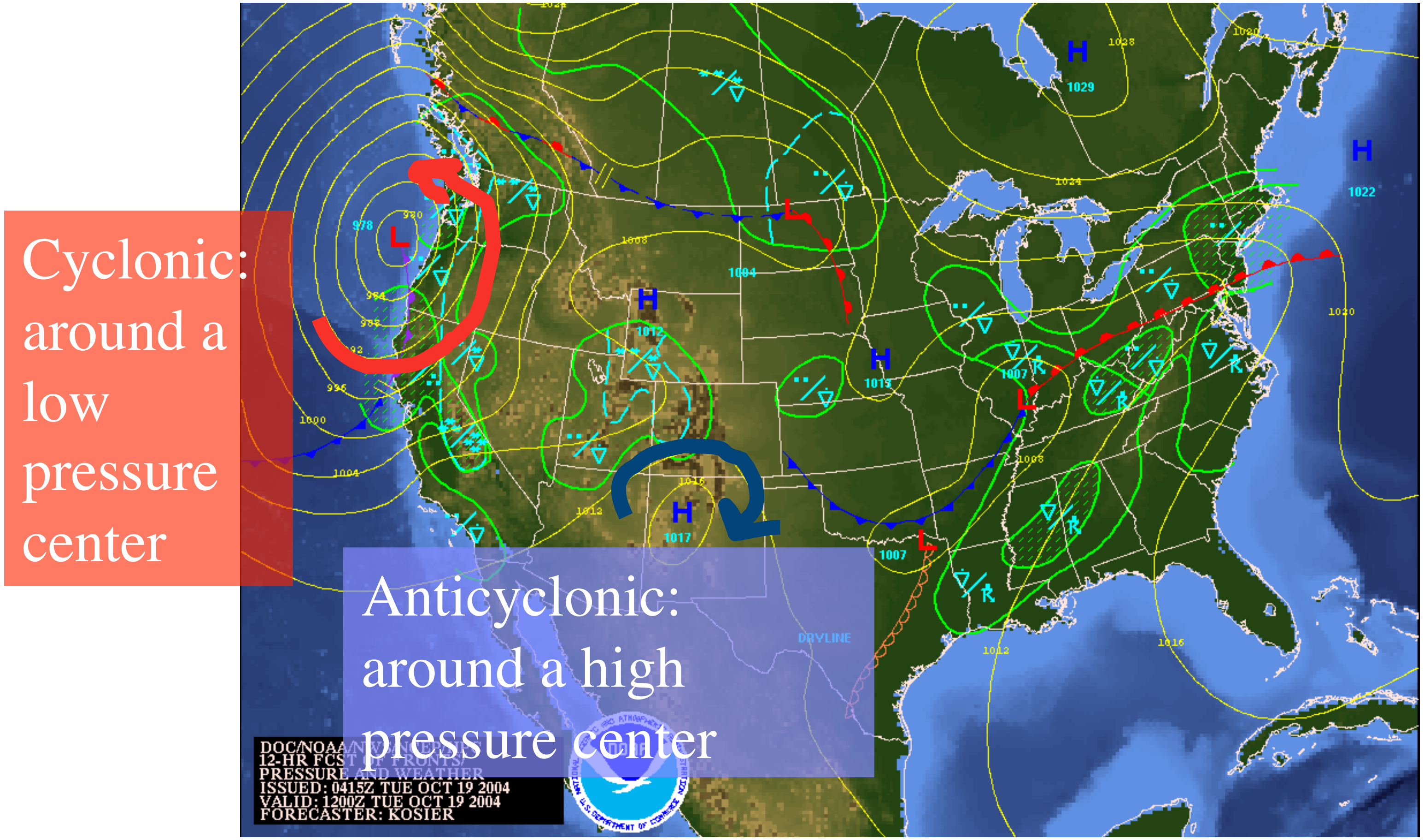
Vertical (z) (down-up)

0 = pressure gradient force + effective gravity

$$0 = -(1/\rho)\partial p/\partial z - g$$

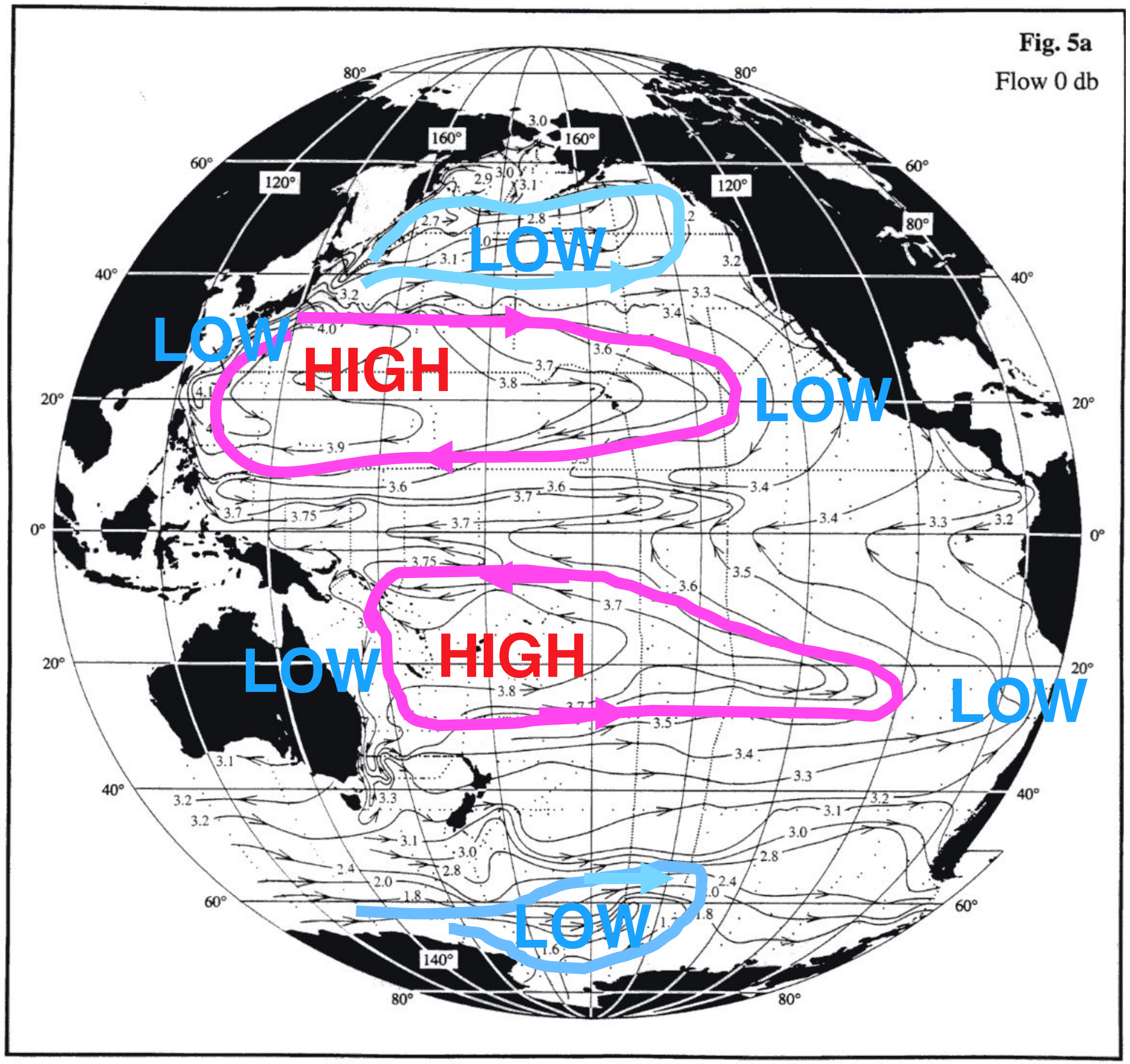
Definitions: Cyclonic and Anticyclonic

NWS daily weather map



Cyclonic is counterclockwise in the northern hemisphere, clockwise in the southern hemisphere

Example from ocean of geostrophic flow
Surface height/pressure and surface geostrophic circulation



Circulation is counterclockwise around the Low (cyclonic) and clockwise around the High (anticyclonic).

(northern hemisphere)

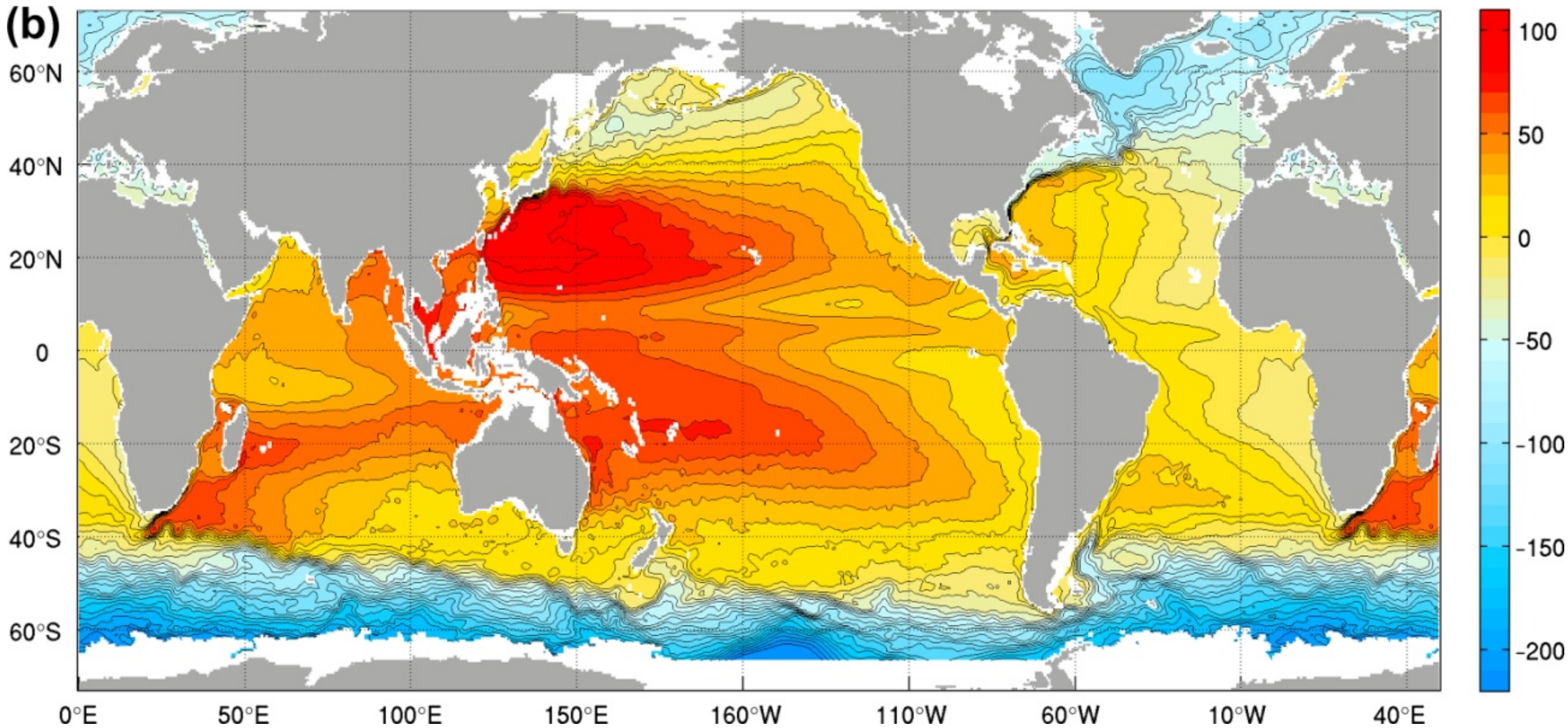
Southern Hem:

Cyclonic is clockwise, etc..

Reid, 1997

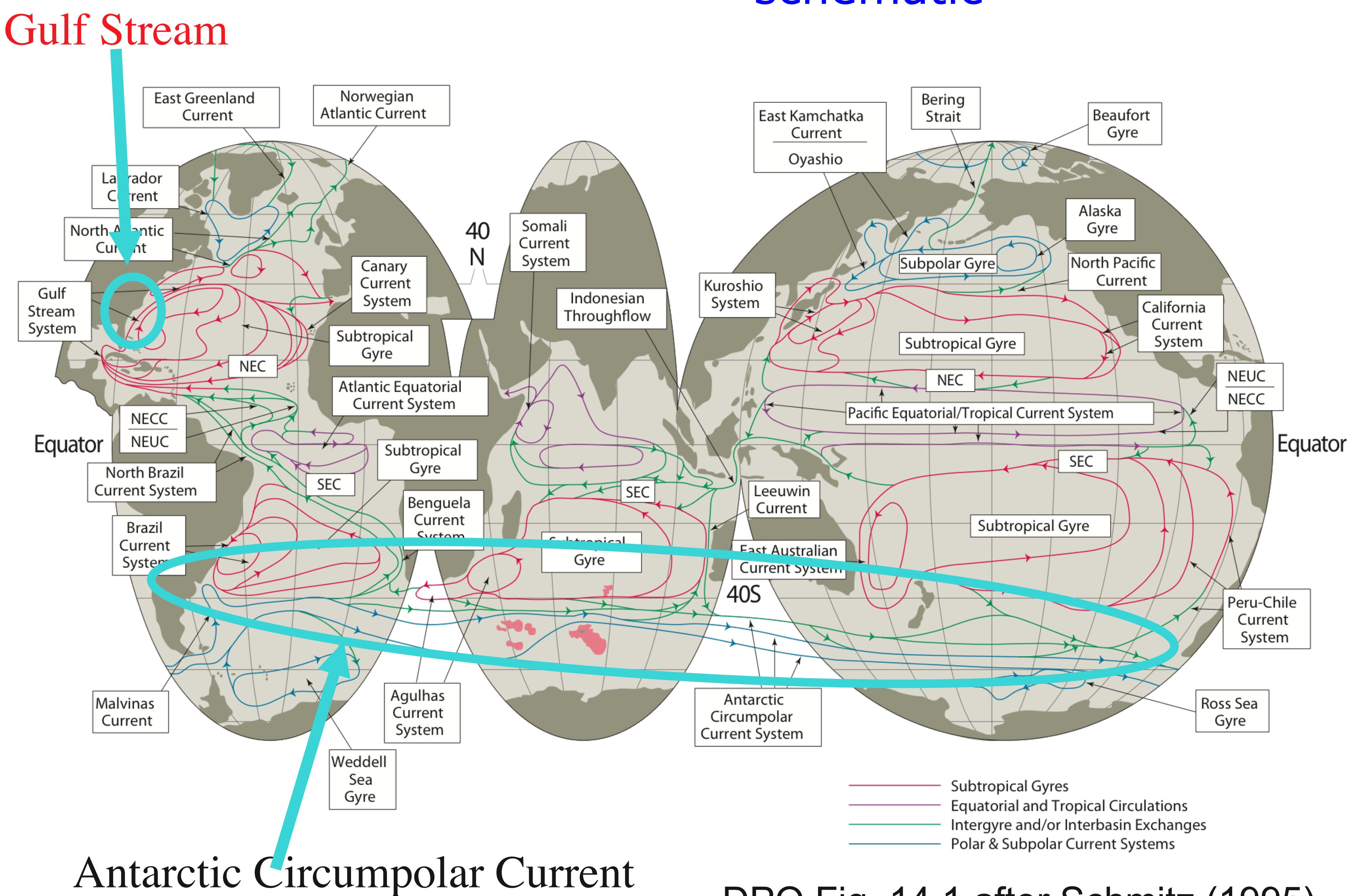
Sea surface height

Compare with surface circulation (next slide)



DPO Fig. 14.2

Surface (geostrophic) circulation schematic



DPO Fig. 14.1 after Schmitz (1995)

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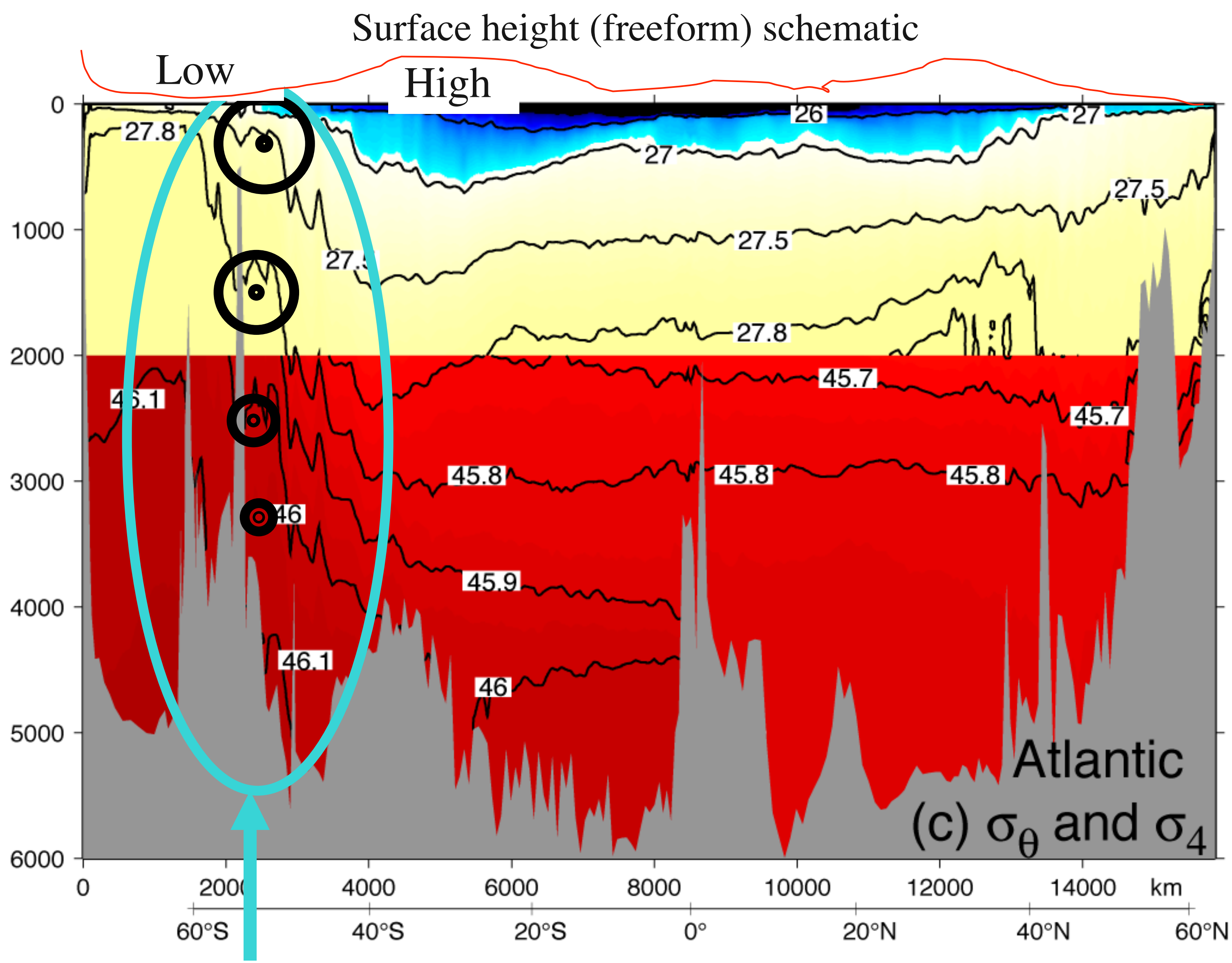
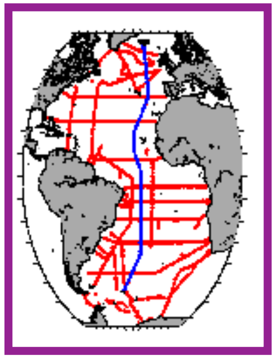
Vertical structure of the circulation (Atlantic section of potential density)





Antarctic Circumpolar Current

Talley SIO 210 (2015)

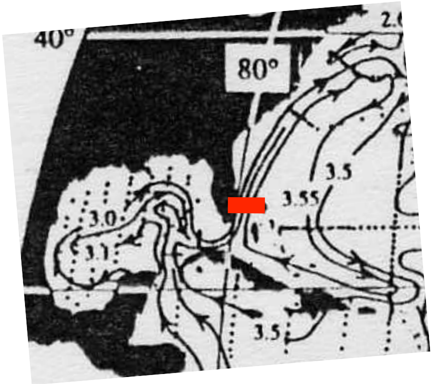
DPO Fig. 4.10

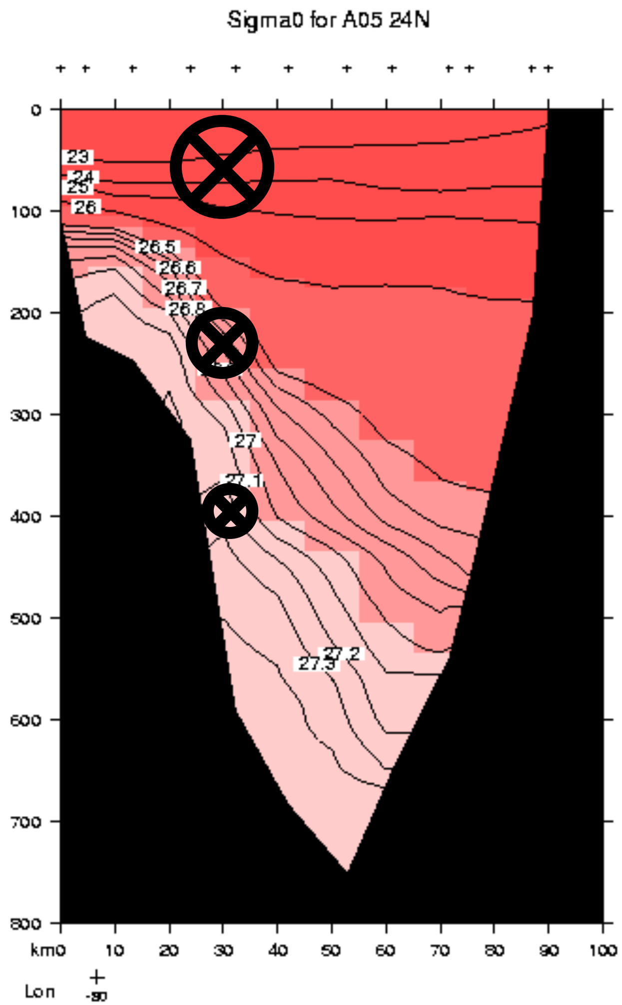
To observe geostrophic circulation

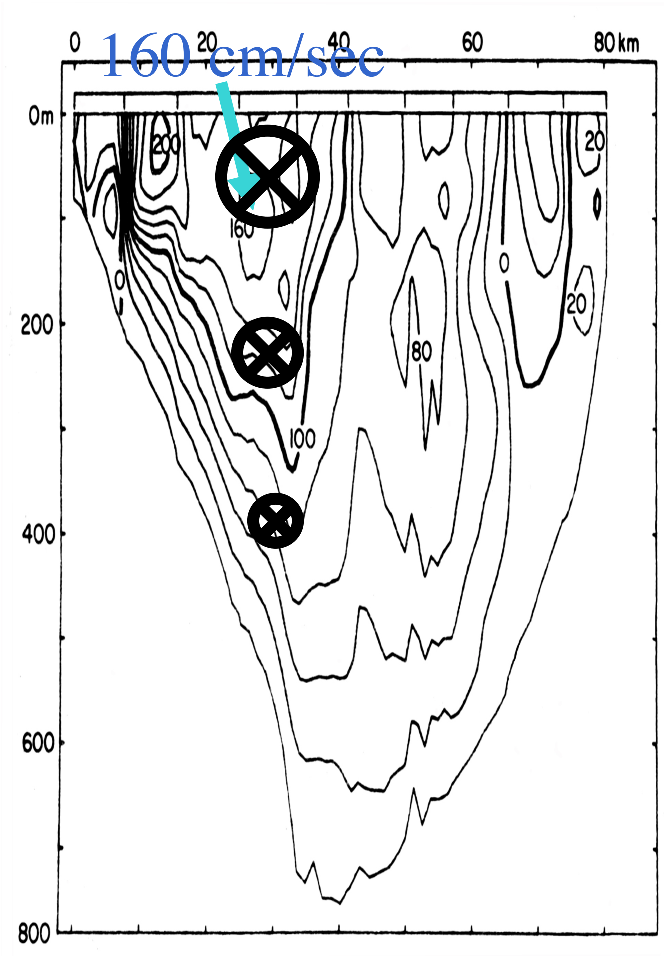
- WE WANT: Current speed and direction
- WE MEASURE: Density (temperature, salinity)
- We can’t measure the sea surface height accurately enough (using altimeters)

- Steady-state isopycnal slopes tell us that the geostrophic current is varying with depth (“vertically sheared”)
- Most currents we see are wind-driven and therefore strongest at the sea surface, decaying with depth
- We therefore make a very educated (or very uneducated guess) at the current speed and direction at some depth, and use the density field to figure out how the current changes with depth AT THAT LOCATION in latitude and longitude
- Another example - Gulf Stream ----->>

Gulf Stream vertical sections in Florida Strait (west on left)



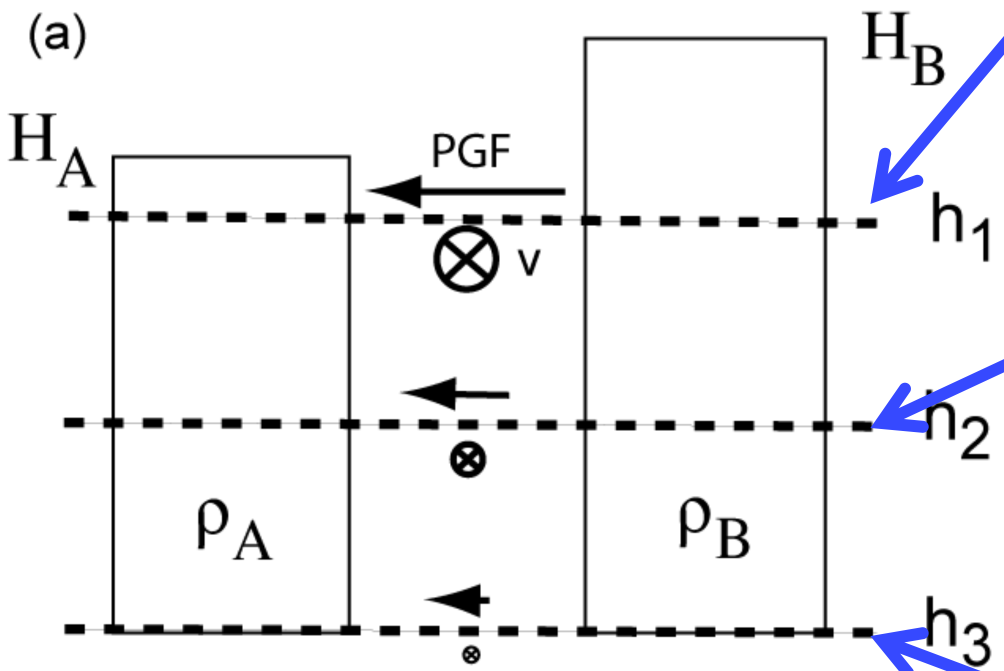




Roemmich, 1983

Thermal wind ("geostrophic method"): How does the geostrophic flow change with depth (below the sea surface)?

Example 1: 2 columns of different density and different height



Near the sea surface (at h_1), there is more mass at B than at A, so there is higher pressure at B than at A, so there is a westward PGF and northward flow (northern hemisphere).

Farther down (at h_2), the difference in mass above h_2 at B and A is smaller since the density at B is lower than an A. So the pgf is smaller and the current is weaker.

Even farther down (at h_3), we can have equal mass (equal pressure, hence NO pgf) since total mass at B and A is the SAME.

For this example: density at A is higher than density at B

$\rho_A > \rho_B$

DPO Fig. S7.20

Thermal wind ("geostrophic method"):
Example 2: continuous density variation and uneven height

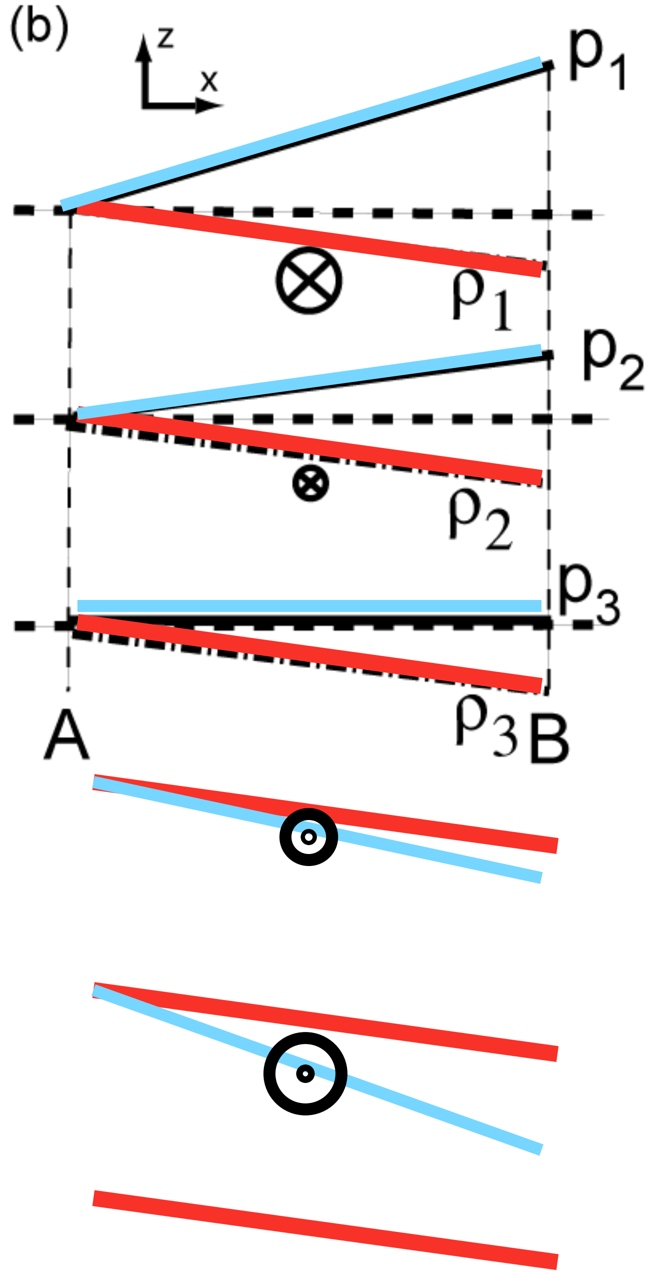
We measure density (RED lines).

Here we assume (or measure) that the pgf and therefore the velocity is 0 at depth 3. Therefore contour p_3 (BLUE) must be flat there.

We use hydrostatic balance to calculate pressure (BLUE lines) at all other depths relative to level 3.

Water is less dense at B than at A, so isobars are higher there, above level 3, than at A. Sea surface is therefore higher at B than at A.

(And below level 3, the flow would start going back the other way, out of the page)



DPO Fig. 7.10

Dynamic height and geopotential anomaly: proxies for pressure

The pgf is calculated as the difference of pressure between two stations at a given depth (flat surface relative to the geoid).

a) If the velocity is known at a given depth, then the pgf at that depth is also known (from geostrophy)

b) From the measured density profiles at the two stations, we can calculate how the pgf changes with depth, which tells us the velocity change with depth.

c) Using the known velocity from (a), which we call the reference velocity, and knowing how velocity changes with depth from (b), we can compute velocity at every depth.

Thermal wind (formal expressions)

The vertical shear in geostrophic velocity is proportional to the horizontal density derivative.

Formally:

$$- f\partial v/\partial z = (g/\rho_o)\partial\rho/\partial x$$

$$f\partial u/\partial z = (g/\rho_o)\partial\rho/\partial y$$

(Section 2: this is derived from the geostrophic x, y momentum balances (take the vertical derivative) and then substituting from hydrostatic balance for the vertical pressure derivative.)

Geopotential anomaly and dynamic height

To get the change in pgf from depth 1 to 2, get the mass of water on both profiles between depths 1 and 2. Historically use the specific volume anomaly δ, which is $\delta = \alpha - \alpha(35, 0°, p)$ where $\alpha = 1/\rho$ is specific volume. Then choose/measure/guess the reference velocity at, say, depth 1 and compute velocity at depth 2

$\Delta\Phi = -\int \delta \, dp$ geopotential anomaly

$f(v_2 - v_1) = -\partial \Delta\Phi/\partial x$ (8.26a)

$f(u_2 - u_1) = \partial \Delta\Phi/\partial y$ (8.26b)

OR

$\Delta D = -\Delta\Phi / 10 = -\int \delta \, dp \, /10$ dynamic height

$1 \text{ dyn m} = 10 \text{ m}^2/\text{sec}^2$

$f(v_2 - v_1) = 10 \, \partial \Delta D/\partial x$ (8.26a)

$f(u_2 - u_1) = -10 \, \partial \Delta D/\partial y$ (8.26b)

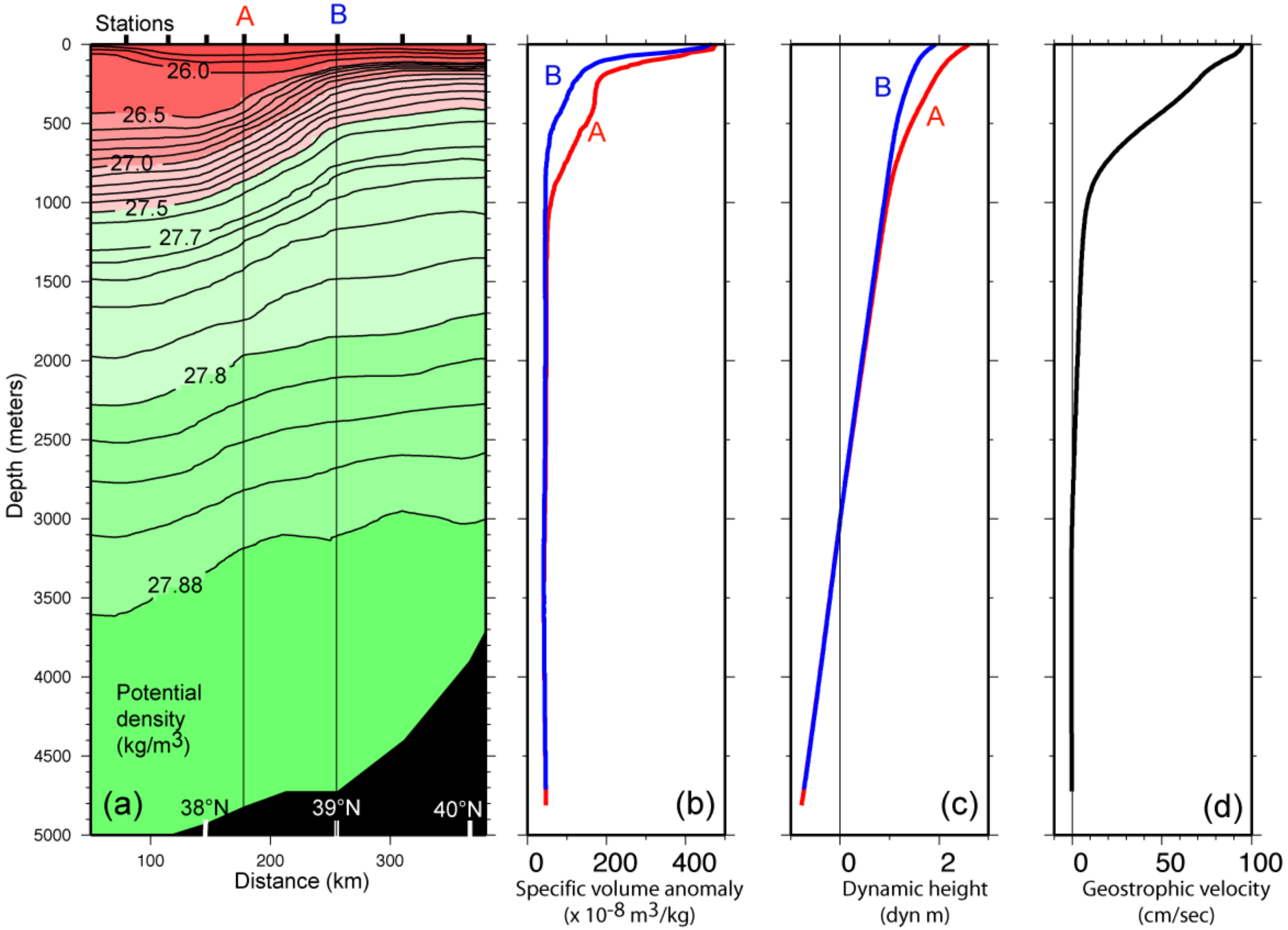
Geopotential anomaly and dynamic height

Reference velocities:

Level of no motion: Old-fashioned - assume 0 velocity at some great depth, and compute velocities at all shallower depths. This is useful if you are focused on very energetic upper ocean flows, but not useful if you want to look at small deep flows.

Level of known motion: Much better - observe or determine through external means (use of tracers, mass balance, etc) a good guess at the velocity at some depth. This is essential if you want to study deep circulation.

Gulf Stream density, dynamic height and geostrophic velocity

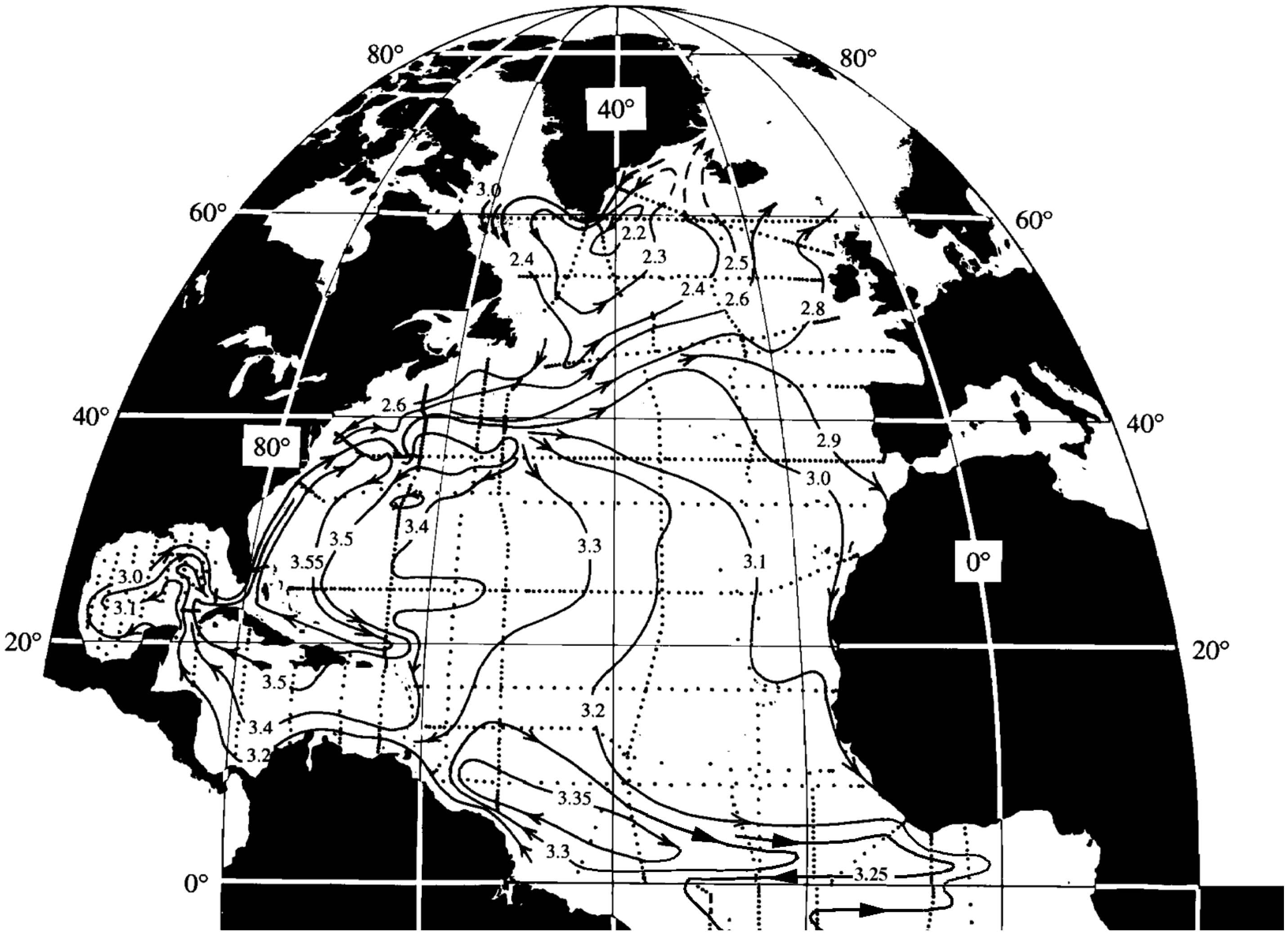


In this example, compute geostrophic velocities relative to 0 cm/sec at 3000 dbar. If we KNOW v at 3000 dbar, then just add it to the whole profile

DPO Fig. 7.11

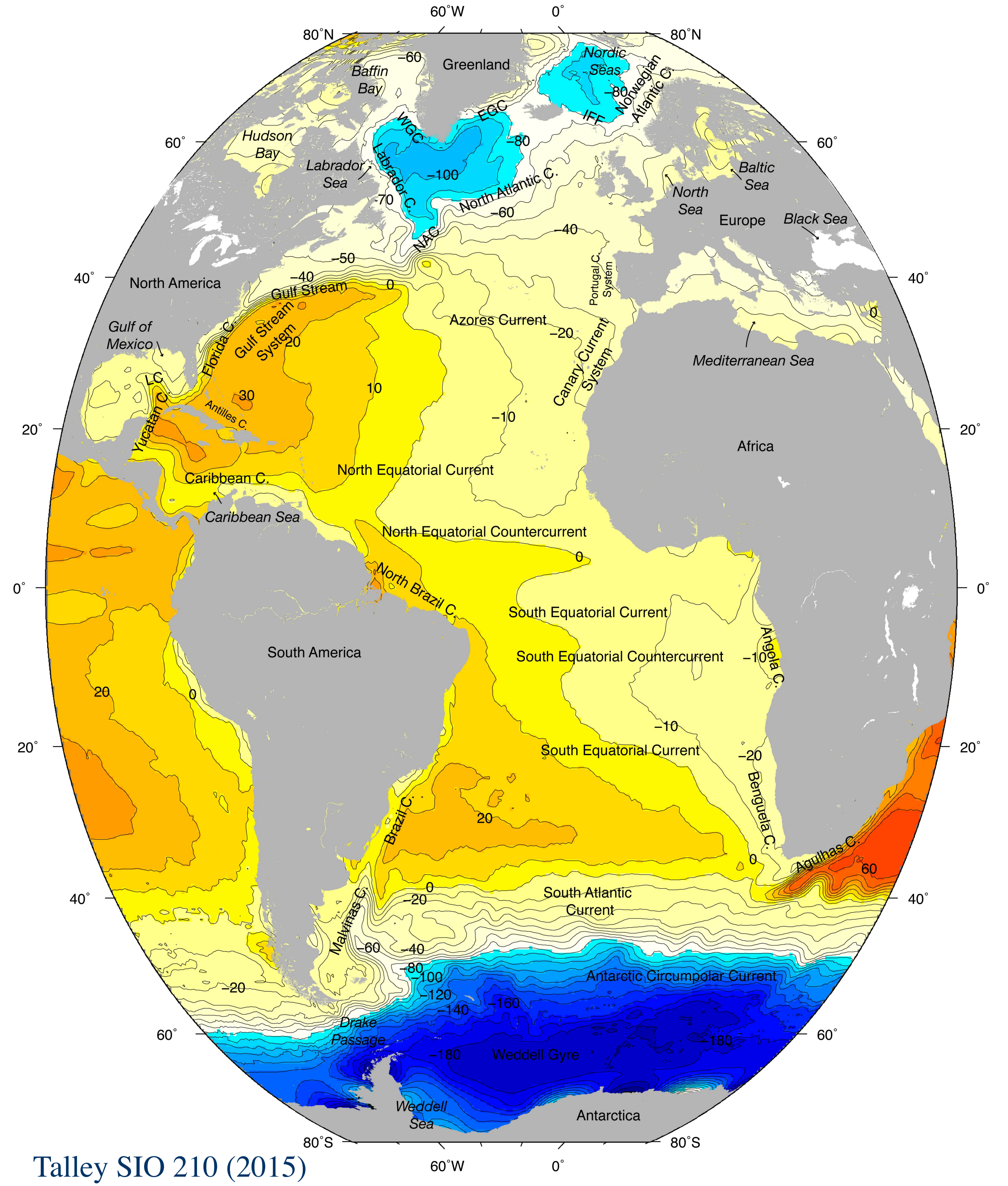
Steric height at the sea surface in the N. Atlantic.

Values are similar to sea surface height in meters. (Reid, 1994)



DPO Fig. 9.2a

Sea surface height in the Atlantic. (Using Niiler et al., 2003, surface heights based on drifters)



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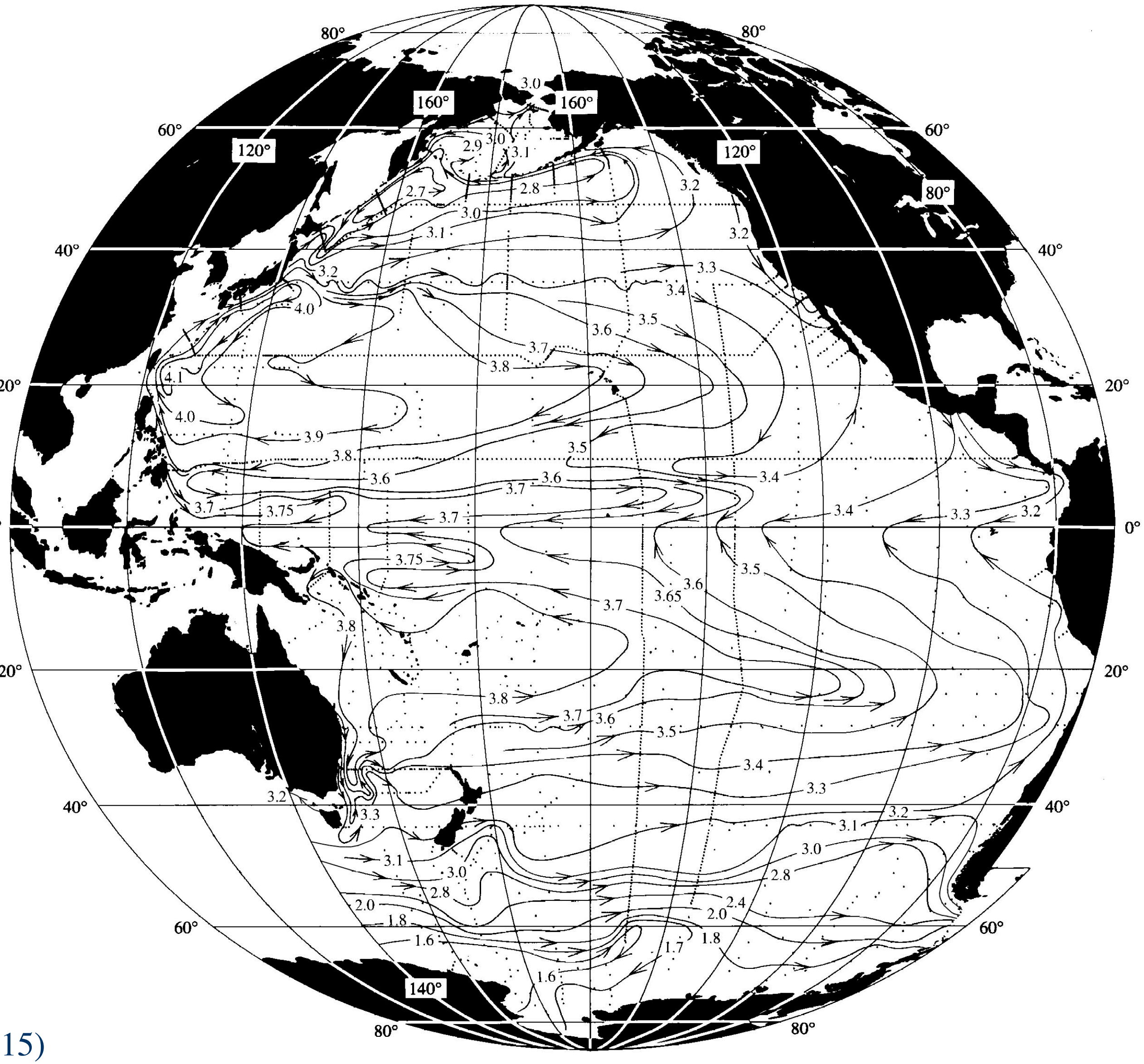
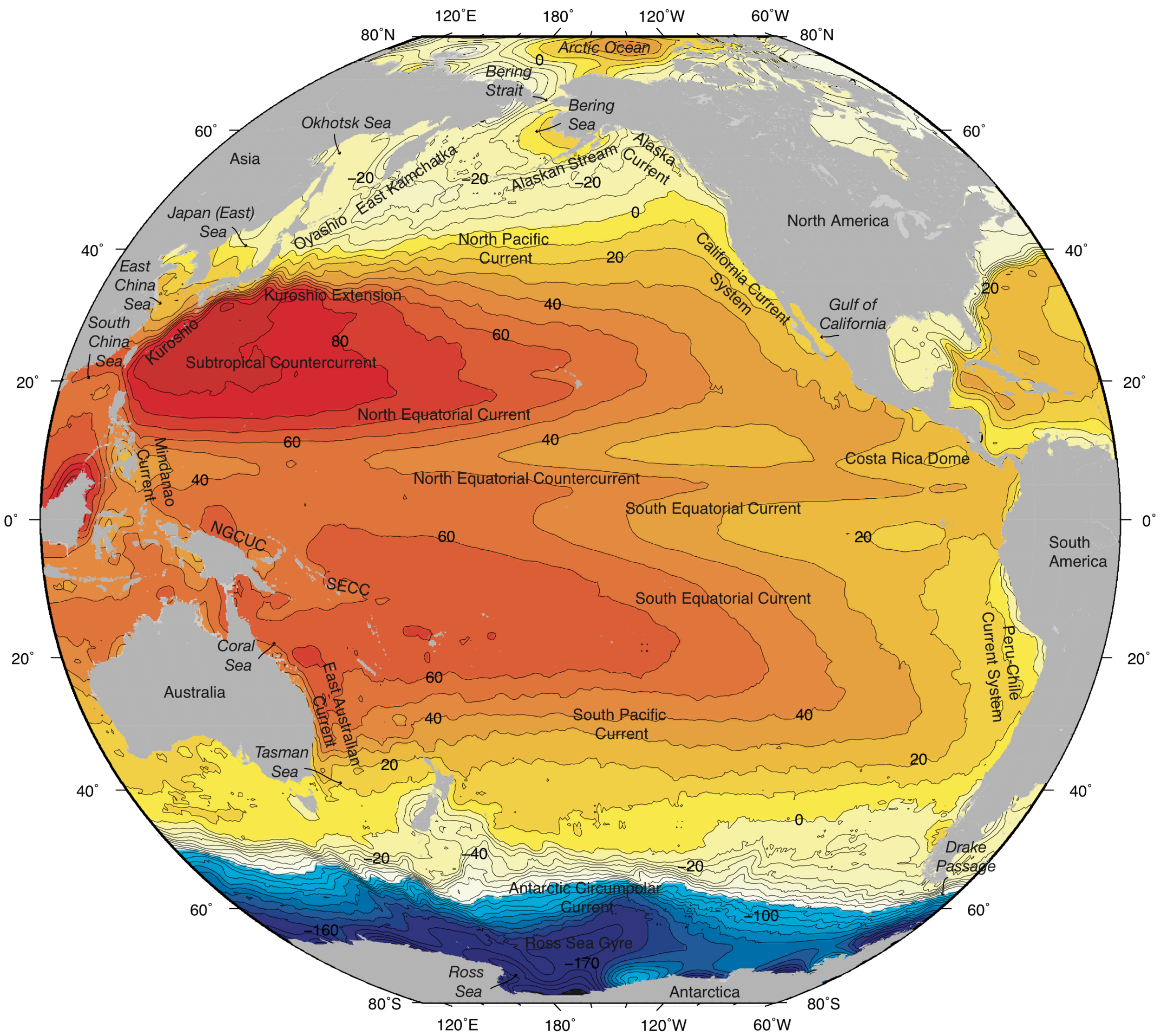
DPO Fig. S9.1

Steric height at the sea surface in the Pacific. Values are similar to sea surface height in meters. (Reid, 1997)



DPO Fig. 10.2a

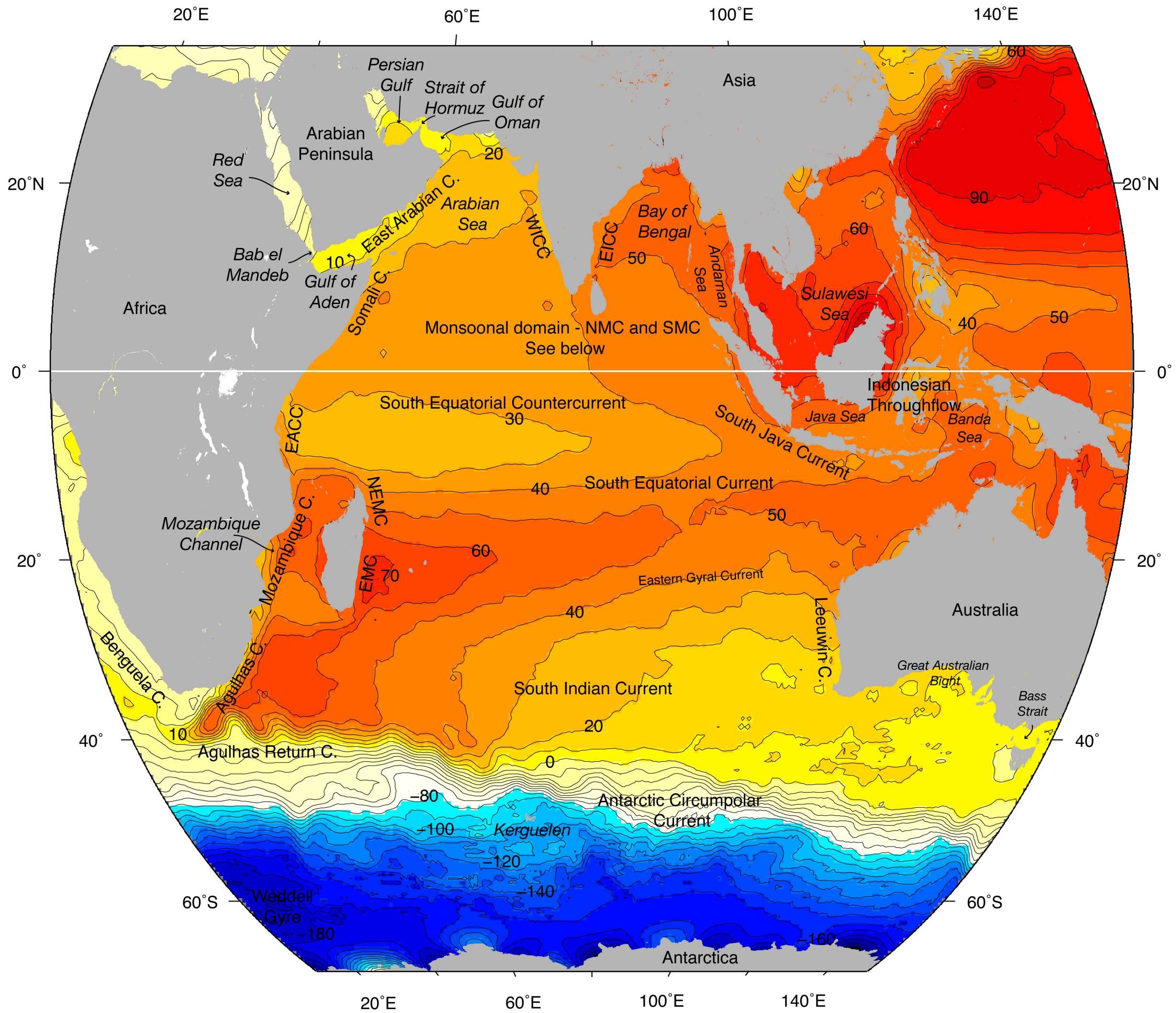
Sea surface height in the Pacific. (Using Niiler et al., 2003, surface heights based on drifters)



DPO Fig. S10.1

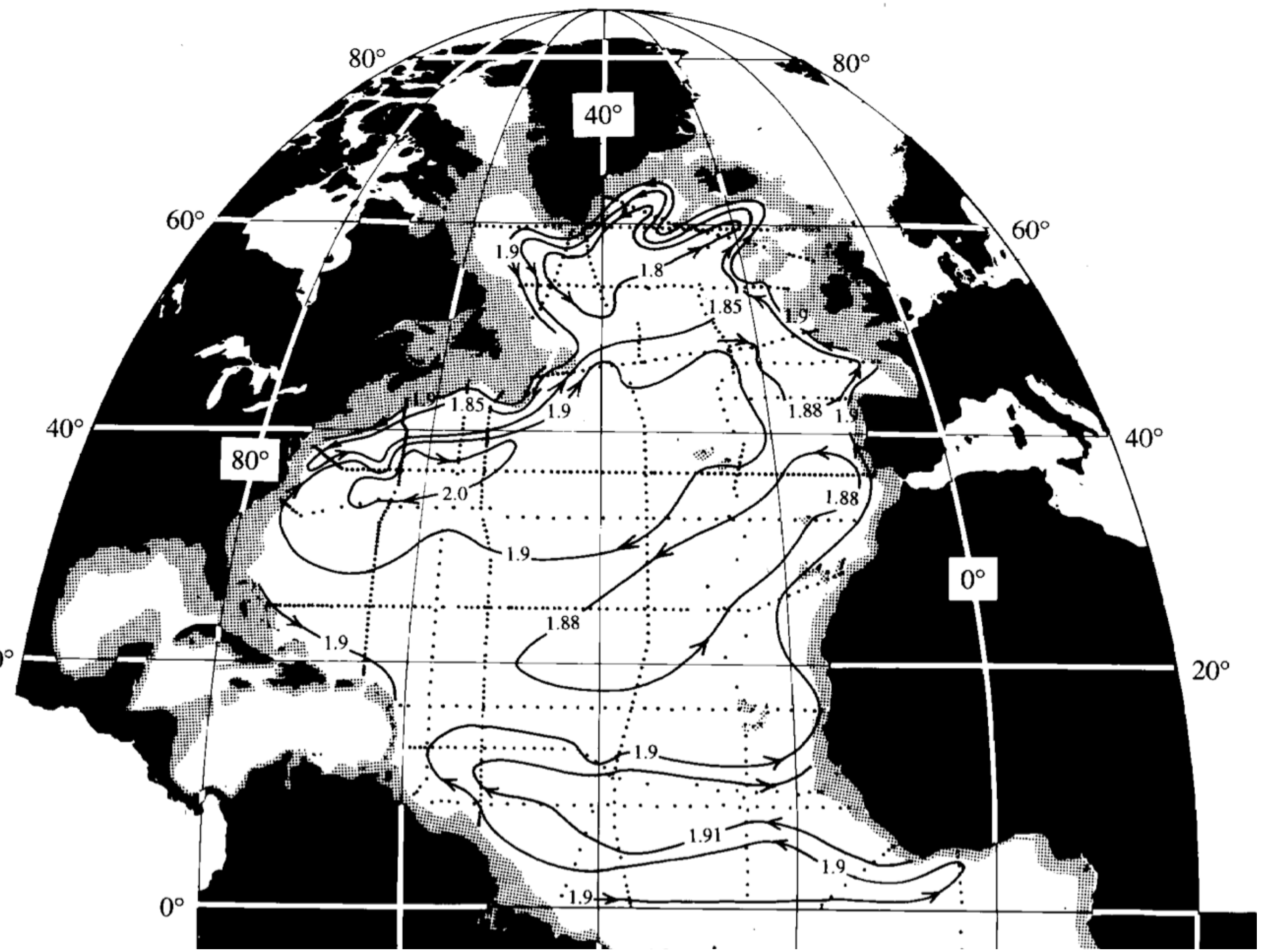
Talley SIO 210 (2015)

Sea surface height in the Indian. (Using Niiler et al., 2003, surface heights based on drifters)



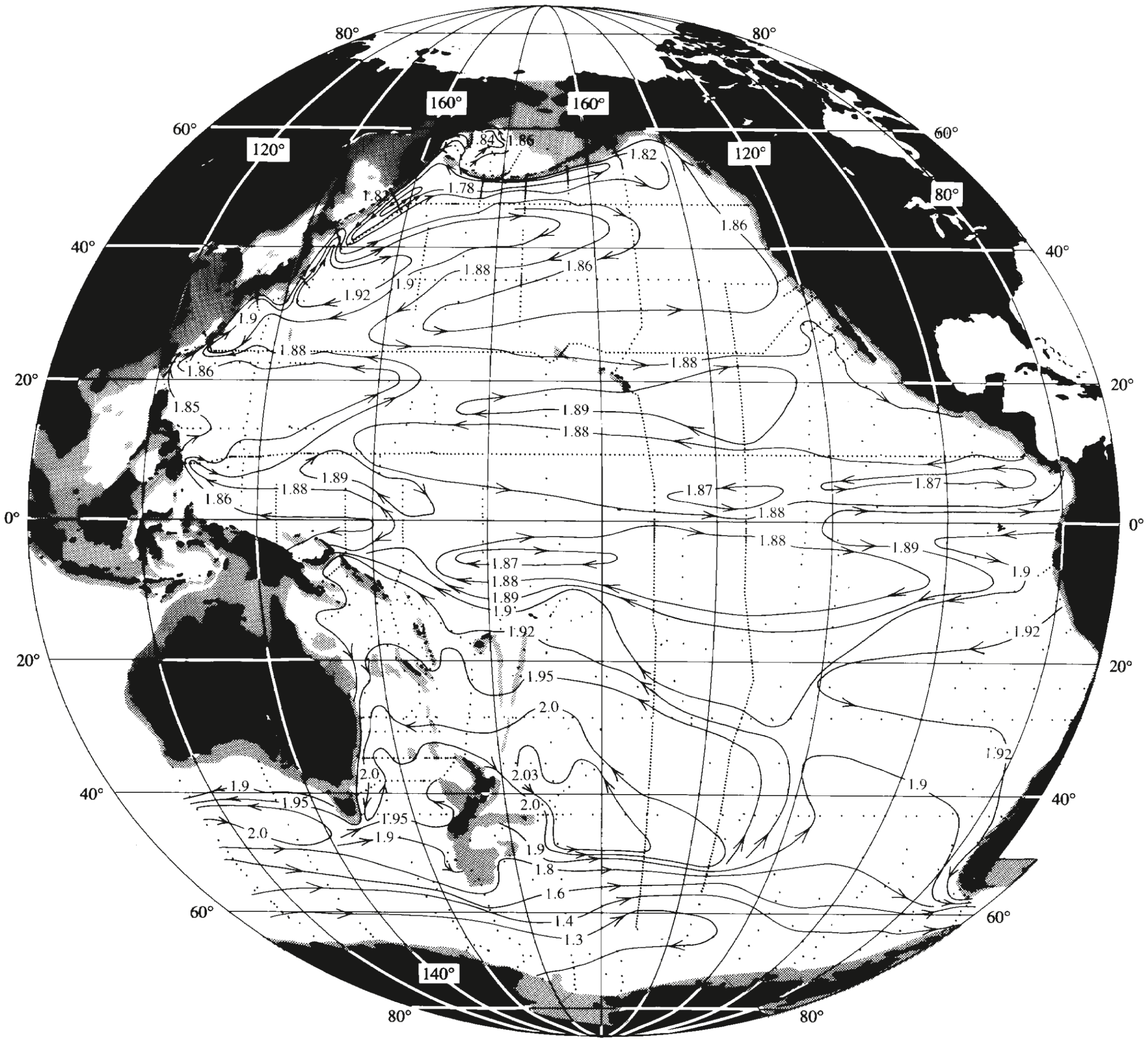
DPO Fig. S11.1

Steric height at 1000 dbar in the N. Atlantic. Values are similar to sea surface height in meters. (Reid, 1994)



DPO Fig. S9.2b

Steric height at 1000 dbar in the Pacific. Values are similar to sea surface height in meters. (Reid, 1997)



DPO Fig. 10.10